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Strongly frustrated random antiferromagnets

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Abstract. A high dimensionality calculation (Weiss like) has been carried out for antiferromagnetism (AFM) in structures with many sublattices. By allowing quenched disorder in the exchange interactions our results clearly exhibit the interplay between the effects of lattice frustration and disorder on the system's properties. For given number of sublattices present, there are several possible phases (ordering of the spins) and as many metastable states in the ergodic phases. It is found that the glassy behavior, and metastability, for multi-sublattices systems is substantially enhanced as compared with simple structures, exhibiting structure dependent de Almeida-Thouless lines. Strongly disordered systems have the long-range AFM ordering, ergodic metastable states and glassy phases intermingled in a non-trivial way. Also, even small fluctuations in the exchange parameters do induce sizeable glassy behavior in structures with many sublattices. Spin glass behavior in apparently non-disordered systems as certain pyrochlores may be accounted for within the present context.

PACS. 75.40.Cx Static properties (order parameter, static susceptibility, heat capacities, critical exponents, etc.) – 75.10.Nr Spin-glass and other random models – 75.50.Lk Spin glasses and other random magnets

1 Introduction

Magnetic order in systems with complicated lattices structures has been intensely studied recently [1]. In such systems, as for example $Tb_2Mo_2O_7$, strong geometrical frustration may be present in the interactions among the spins. Even for purely ferromagnetic interactions, lattice constraints may originate frustration effects in interacting Heisenberg spins [2]. The ground state and low-lying excitations of such systems is the subject of current interest and largely unknown, notably the glass-like (meaning spin-glass like) behavior in apparently non-disordered systems [1–3]. In addition, the understanding of phase transitions in random field magnets (or from the experimental viewpoint diluted antiferromagnets in an external field) in finite dimension (d) remains an open problem as recent renormalization group calculation by Brézin and de Dominicis [4], within a ϕ^4 theory, has shown; in close analogy with spin glasses [5,6] no stable fixed points are found for $d < 8$, leaving room to several speculations as to the correct way to treat analytically such systems encoding frustration and some sort of disorder in relevant finite d: no known perturbation scheme seems to work. In such scenario, it may be helpful to have a consistent good qualitative view of the phenomena involved though it may be not unique [7] for the shortcomings in the models cannot be easily assessed [8].

In the present work we introduce an Ising model which is good for multi-sublattices antiferromagnetic systems, allowing quenched disorder in the interactions. As Anderson pointed out long ago [9], the simple Néel theory for antiferromagnetism with two-sublattices is not applicable to most lattice structures encountered in actual antiferromagnets, a better agreement with experimental results being achieved taking into account lattice structure. The model here studied is basically a generalization of Néel's molecular field theory and that of the Sherrington-Kirkpatrick (SK) model [10] (both valid for d large). It incorporates strong frustration effects among the sublattices spins through a mean pure antiferromagnetic interaction J_0, competing with random interactions which also gives rise to frustration and glassiness. In a field the transitions from the paramagnetic to the antiferromagnetic phase or within the various ordered phases may be continuous (second order) or discontinuous (first order) like when metamagnet effects are included as is well-known at least for the two-sublattices case [11,12]; here for simplicity we shall study only the case of same mean antiferromagnetic intersublattice interactions and the general features of the phase diagrams. For more than two-sublattices, there is a multicritical line separating the paramagnetic and antiferromagnetic phases at zero external field (h) The temperature at which irreversibility sets in, as function of the external field or J_0 (de Almeida-Thouless (AT) lines) depends on the number of sublattices (p) considered with the non-ergodic phases growing in size as p increases. For multi-sublattices ($p \geq 3$) ordering systems,

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there is an asymmetry in the phase diagram in zero field under the operation $J_0 \to -J_0$. The glassy behavior is greatly enhanced as compared with simple structures thus suggesting that small fluctuations in the exchange interactions may induce glassy behavior even in non-disordered systems. Metastable states are always present in the ergodic ordered phases (a fact overlooked in previous study for the two-sublattices case [12]) which are locally stable over appreciable regions in the phase diagram, there arising the possibility of the system going into a glassy phase from an ergodic metastable state. In addition, if it turns out that the present modeling of physical systems is correct, the distinct shapes of the AT's lines for the different spin ordering structures in antiferromagnetic systems with multi-sublattices may serve the purpose of characterizing spin ordering in complex structures.

2 The model

Following Edwards and Anderson [13] the Hamiltonian for a short-range Ising model on a regular lattice may be written

$$H = -\sum_{(ij)} J_{ij}\sigma_i\sigma_j - h\sum_i \sigma_i, \quad \sigma_i = \pm 1 \tag{1}$$

where the J_{ij} are taken to be random variables with a distribution dependent on the lattice separation between sites i and j, with mean and variance J_0 and J^2, respectively. Here we want to study the mean field limit (mfa) of (1) when the system of interest may be subdivided into many sublattices like in a fcc lattice but every spin seeing only spins on the others sublattices. In this case, to get sensible results the long-range model must accordingly allow the possibility of the various ordering among the sublattices as in Anderson [9]. Thus, for a long-range mfa version of (1) we consider the Hamiltonian

$$H = -\sum_{(\mu\nu)}\sum_{i,j=1}^{N} J_{ij}^{\mu\nu}\sigma_{i\mu}\sigma_{j\nu} - h\sum_{\mu=1}^{p}\sum_{i=1}^{N}\sigma_{i\mu} \tag{2}$$

where $i,j = 1,2,3,\ldots,N$ number the sites on each sublattice $\mu = 1,2,\ldots,p$; $(\mu\nu)$ meaning the distinct pairs of sublattices; h is an external magnetic field. The exchange interactions $J_{ij}^{\mu\nu}$ are assumed distributed according to a Gaussian

$$P(J_{ij}^{\mu\nu}) = \Big(2\pi(J^{\mu\nu})^2/N\Big)^{-1/2} \times \exp\big[-N(J_{ij}^{\mu\nu} + J_0^{\mu\nu}/N)^2/2J^2\big] \tag{3}$$

and for simplicity we shall assume a uniform mean and variance, $J_0^{\mu\nu} = J_0/(p-1)$ and $(J^{\mu\nu})^2 = J^2/(p-1)$, respectively. All energies shall be measured in units of J. For studying pure antiferromagnetism ($J = 0$) in complex lattice structures this model is more appropriated than the simple Néel two-sublattices approach [9]. Weiss's molecular field theory for ferromagnetism is recovered for $J_0 < 0$, $p = 2$, $J = 0$ while Néel's theory is obtained for $J_0 > 0$, $p = 2$, $J = 0$. For $J_0 > 0$, $J = 0$ and p very large the model falls in the category of fully frustrated models [14]. It is a generalization of the SK [10] model. Following standard procedure the free energy per spin within the replica approach is given by

$$f = -\frac{\beta J^2}{4} + \frac{1}{p\beta}\lim_{n\to 0}\frac{1}{n}\Phi\{m_p^\alpha; q_p^{\alpha\beta}\} \tag{4a}$$

where

$$\begin{aligned}\Phi\{m_p^\alpha; q_p^{\alpha\beta}\} = &-\frac{\beta J_0}{(p-1)}\sum_{(\mu\nu)}\sum_{\alpha=1}^{n} m_\mu^\alpha m_\nu^\alpha \\ &+ \frac{\beta^2 J^2}{(p-1)}\sum_{(\mu\nu)}\sum_{(\alpha\beta)}\left(1 - q_\mu^{\alpha\beta}\right)\left(1 - q_\nu^{\alpha\beta}\right) \\ &- \ln Tr\exp\Bigg[-\frac{\beta J_0}{(p-1)}\sum_{(\mu\nu)}\sum_{\alpha} m_\mu^\alpha \sigma_\nu^\alpha \\ &+ \frac{\beta^2 J^2}{(p-1)}\sum_{(\mu\nu)}\sum_{(\alpha\beta)} q_\mu^{\alpha\beta}\sigma_\nu^\alpha\sigma_\nu^\beta \\ &+ \beta h\sum_\mu\sum_\alpha \sigma_\mu^\alpha\Bigg]\end{aligned} \tag{4b}$$

where $\alpha,\beta = 1,2,\ldots,n$ are replica indices and m_μ^α, $q_\mu^{\alpha\beta}$ are variational parameters associated to the sublattices magnetization and spin glass order parameters. As usual, to explore the thermodynamics and possible ordering in the case of multi-sublattices antiferromagnetic systems the first ansatz to solve (4) is suppose a replica symmetric solution $m_\mu^\alpha = m_\mu$, $q_\mu^{\alpha\beta} = q_\mu$ together with the study of fluctuations around this solution [8,10,15]. The replica symmetric solution for the free energy per spin, from (4), is

$$\begin{aligned}f = &-\frac{J_0}{p(p-1)}\sum_{(\mu\nu)} m_\mu m_\nu - \frac{\beta J^2}{2p(p-1)}\sum_{(\mu\nu)}(1-q_\mu)(1-q_\nu) \\ &- \frac{1}{p\beta}\int\cdots\int\left(\prod_{\mu=1}^{p} Dz_\mu\right)\ln\left[2^P\prod_{\mu=1}^{p}\cosh(E_\mu)\right]\end{aligned} \tag{5}$$

where

$$E_\mu = \beta\Bigg(h - \big(J_0/(p-1)\big)\sum_{\nu(\neq\mu)} m_\nu + z\beta J \times\left[\sum_{\nu(\neq\mu)} q_\nu/(p-1)\right]^{1/2}\Bigg) \tag{6a}$$

$$Dz = (2\pi)^{-1/2}\exp\left(-z^2/2\right)\mathrm{d}z \tag{6b}$$

$$m_\mu = \int Dz\tanh(E_\mu); \quad q_\mu = \int Dz\tanh^2(E_\mu). \tag{6c}$$

Study of the fluctuations of (4) around the replica symmetric ansatz (6) leads to the eigenvalues whose zeros may yield the replica symmetric phases diagrams and to the AT's lines. The boundaries of the continuous transitions between the possible several phases are given by solutions of the equations (points where pairs of sublattices magnetizations coalesce or some of the staggered susceptibilities diverge)

$$1 = \beta\big(J_0/(p-1)\big)(1-q_\mu) - \frac{2\beta^2\Big(J^2/(p-1)\big(J_0/(p-1)\big)\Big)\Big(m_\mu - q_\mu^{(3)}\Big)^2}{1+\beta^2\big(J^2/(p-1)\big)\Big(1-4q_\mu+3q_\mu^{(4)}\Big)}$$
$$\mu = 1,2,\ldots,p. \tag{7a}$$

$$q_\mu^{(k)} = \int Dz \tanh^k(E_\mu) \quad k=2,3,\ldots \tag{7b}$$

while fluctuations against replica symmetry leads to the following set of eigenvalues for stability against replica symmetry (replicon sector)

$$\lambda_\mu^{(1)} = 1; \qquad \mu = 1,2,\ldots,p \tag{8a}$$

$$\lambda_\mu^{(2)} = 1 + (A_\mu A_\nu)^{1/2}; \quad \mu,\nu = 1,2,\ldots,p \quad (\mu\neq\nu) \tag{8b}$$

$$\lambda_\mu^{(3)} = 1 - (A_\mu A_\nu)^{1/2}; \quad \mu,\nu = 1,2,\ldots,p \quad (\mu\neq\nu) \tag{8c}$$

where

$$A_\mu = \beta^2\big(J^2/(p-1)\big)\Big(1-2q_\mu+q_\mu^{(4)}\Big); \quad \mu = 1,2,\ldots,p. \tag{8d}$$

Comparison of free energies for regions where there are coexisting stable solutions may yield the first order transition lines. The case $p=2$ has partially been worked out before [12]. For continuous transitions onset of glass behavior in the model is given by the condition $\min[\lambda_{\mu\nu}]=0$, which yields the so called de Almeida-Thouless lines. Below these lines broken replica symmetry solution must be enforced, non-ergodic behavior sets in and there occur many metastable states [8]. As might have been anticipated, the replica symmetric solution is unstable over parts of the phase diagram calling into action replica symmetry breaking (RSB). However, even the replica symmetric solution is already enough involved to be obtained for general p (recall that Eqs. (5, 6) may be seem as a set of $2p$ diffusion-like coupled differential equations) with a rich phase diagram and giving insight to the general features of the model. In the next section we explore the numerical solution of equations (5–8). Notice that all replica parameters are interdependent and such that RSB will act simultaneously on all of them. In addition, one is dealing with a parameters space much larger than the SK one for general p and it is not clear that Parisi's RSB prescription should be applied uniformly to all sublattices. For instance, one may or may not associate to each sublattice a double continuum of order parameters $q_\mu(x)$ and $\Delta_\mu(x)$ as in the RSB approach of de Dominicis *et al.* [16]. The SK model solution as found by Parisi corresponds to the gauge $\mathrm{d}[\Delta(x)]/\mathrm{d}x = -x\mathrm{d}[q(x)]/\mathrm{d}x$. Following previous works [8,16–19] it is straightforward to write down the variational RSB free energy functional

$$f = -\frac{\beta J^2}{4} - \frac{J_0}{p(p-1)}\sum_{(\mu\nu)} m_\mu m_\nu - \frac{\beta J^2}{2p(p-1)} \times \left[\int_0^1 \mathrm{d}x \sum_{(\mu\nu)} q_\mu(x)q_\nu(x) - \sum_\mu q_\mu(1)\right] - \int Dz \sum_\mu f_\mu\left(0, \beta\Big(h - \big(J_0/(p-1)\big)\Big)\sum_{\nu(\neq\mu)} m_\nu + z\beta J\sqrt{\sum_{\nu(\neq\mu)} q_\nu(0)/(p-1)}\right) \tag{9a}$$

where

$$m_\mu = \int Dz\varphi_\mu\left(0, \beta\Big(h - J_0/(p-1)\Big)\sum_{\nu(\neq\mu)} m_\nu + z\beta J\sqrt{\sum_{\nu(\neq\mu)} q_\nu(0)/(p-1)}\right) \tag{9b}$$

$$q_\mu(x) = \int Dz\psi_\mu\left(0, \beta\Big(h - J_0/(p-1)\Big)\sum_{\nu(\neq\mu)} m_\nu + z\beta J\sqrt{\sum_{\nu(\neq\mu)} q_\nu(0)/(p-1)}\right). \tag{9c}$$

The functions $f_\mu(x,h)$, $\varphi_\mu(x,h)$ and $\psi_\mu(x,h)$, satisfy the equations ($\tilde{q}_\mu = \sum_{\nu\neq\mu} q_\nu$)

$$\partial_x f_\mu = -\frac{\beta^2 J^2}{2(p-1)}\frac{\mathrm{d}\tilde{q}_\mu}{\mathrm{d}x}\left[\partial_h^2 f_\mu + x(\partial_h f_\mu)^2\right] \tag{9d}$$

$$\partial_x \varphi_\mu = -\frac{\beta^2 J^2}{2(p-1)}\frac{\mathrm{d}\tilde{q}_\mu}{\mathrm{d}x}\left[\partial_h^2 \varphi_\mu + x\partial_h f_\mu \partial_h \varphi_\mu\right] \tag{9e}$$

$$\partial_x \psi_\mu = -\frac{\beta^2 J^2}{2(p-1)}\frac{\mathrm{d}\tilde{q}_\mu}{\mathrm{d}x}\left[\partial_h^2 \psi_\mu + x\partial_h f_\mu \partial_h \psi_\mu\right] \tag{9f}$$

with the boundary conditions $f_\mu(1,h) = \ln\big(2\cosh(\beta h)\big)$, $\varphi_\mu(1,h) = \tanh(\beta h)$ and $\psi_\mu = (x,h) = \varphi^2(x,h)$ and assuming same RSB protocol to all sublattices. It is not known whether in the model here considered a uniform gauge should be used for all sublattices or if distinct gauges would have any physical consequence (this consideration has been overlooked for the case $p=2$). Anyway, the set of equations (9) may be easily rewritten for a generalized gauge context [16,19]. Again, the set of equations (9) can be solved numerically or analytically close to Néel temperature $T_N(h=0)$. Its full solution even for the case $p=2$, as far as the author is aware, has not been carried out and only partial results close to especial points has been explored [12]. Here we shall consider the solutions of equations (5–8). Nonetheless, following previous works considering the two-sublattices antiferromagnetic Random Energy Model (REM) [20] extended for the present context, we find that the RSB associated to ergodic metastable states is not the same RSB associated to the most stable solution. This is in close analogy with p-spins interactions model [21] which have 3 phases: paramagnetic, a spin glass phase with one replica symmetry breaking (like the REM model) and another lower temperature spin glass phase with an infinite number of breakings (similar to the SK model); so, in the model studied in [21], the condensed phases have distinct RSB and thus distinct morphologies. In the present model the same kind of behaviour seems to occur and probably should have been expected, the difference being that here all phases involve an infinite number of RSB thus suggesting a breaking of gauge symmetry which may be associated to the distinct possible sublattices orderings. Only further work will clarify these points.

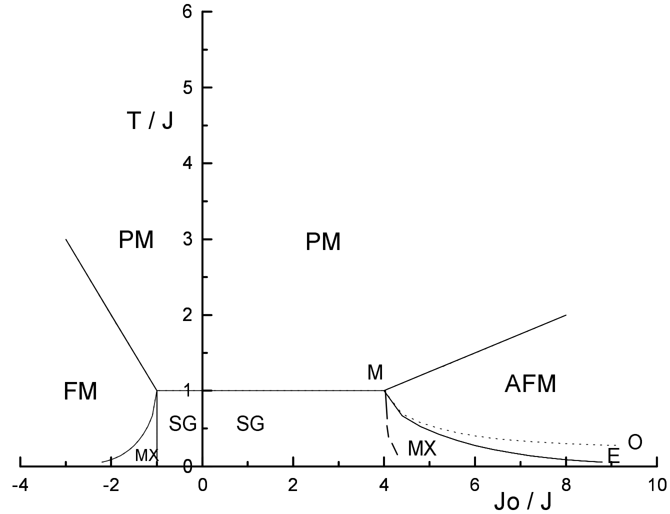

Fig. 1. Zero field phase diagram for the multisublattice antiferromagnetic SK model. It is shown the case $p = 5$. Line segment MO is the instability line for the metastable solution while line ME is for the global minimum of f. All phases are as indicated: PM (paramagnetic), SG (spin glass), MX (mixed phase spin glass-ferromagnetic ($J_0/J < 0$) or mixed spin glass-antiferromagnetic ($J_0/J > 0$)). M is the AFM multicritical point.

3 Results and discussion

The set of equations (5–8) has many possible solutions which depend on the values of (T/J, h/J, J_0/J) and the number of sublattices considered, p .We have solved them numerically for the cases $p = 2, 3, 4$, and 5 and below we give the account of so doing. Our method was to iterate equations (6) starting from low and high temperatures or fields. First, lets us consider the phase diagram when $h = 0$ for which the cases $p = 1$ and 2 has been extensively studied [8,12] and then the finite field case.

3.1 Zero field case

In zero field ($h = 0$) equations (5–7) for one-sublattice (with $J_0 < 0$) or $p = 2$ (with $J_0 > 0$) give the well known SK phase diagram [8,12] where one finds the phases: paramagnetic (PM), antiferromagnetic (AFM), spin glass (SG) and mixed antiferromagnetic-spin glass (MX); it is symmetrical with respect to the cases $J_0 \to -J_0$ (antiferromagnetism/ferromagnetism) for $p = 2$, and Figure 1 would be symmetrical. For $p \geq 3$ there will always be present a lattice frustration among the sublattices; the set of equations (5) almost obviously calls for considering only two cases: p even or p odd. In the case of even p the sublattices spin ordering is one where half of them have magnetization in one direction and the other half in the opposite direction, going back to case $p = 2$ and almost the same phase diagram (but now the line separating the phases PM and AFM is a multicritical line (for $p > 2$) for on turning on a field p phases emerge from it); however, due to the extra intersublattice frustration (geometrical in nature) coming from the finite J_0 the spin-glass phase is increased like shown in Figure 1. For odd p one of the sublattices will be unbalanced with respect to the others which may be grouped ("up" and "down") as in the previous case; this sublattice magnetization is always zero below the Néel temperature but its Edwards-Anderson order parameter is not as can be easily inferred from equations (5) and so the zero field p odd case cannot be mapped to the case $p = 2$ by any rescaling of the parameters. So, there is an asymmetry between the zero field phases diagrams involving ferromagnetism and antiferromagnetism ($p \geq 3$) for the latter case depends on the p value and the symmetry cannot be restored by any rescaling of the parameters. Figure 1 shows the phase diagram for the case $p = 5$; notice that metastable states which appear in the AFM phase may now become unstable against breaking of replica symmetry and this yields the line segment MO while the line ME is the instability line associated to the global minimum of the free energy. Concerning critical exponents, we have checked that both lines ME and MO in Figure 1 approach the multicritical point M with the same AT-line exponent of the SK model, namely $(1 - T/J)^2 \sim (J_0/J - 1)$ [22]. One interesting point to notice is the great increase in the spin glass phase as p is increased: for very large p there will be practically only the phases PM and SG separated by a flat phase boundary given by $T/J = 1$. This corresponds to a fully frustrated antiferromagnet where the fluctuations in the exchange parameters (J) can be arbitrarily small. This kind of behavior has been systematically observed in pyrochlores systems [1,2] finding a natural explanation in the present context: as long as fluctuations in the exchange interactions are present (from thermal, quantum or other nature) and if they may be considered quenched on the time scale of the measurements, there will be a spin glass phase at low temperatures.

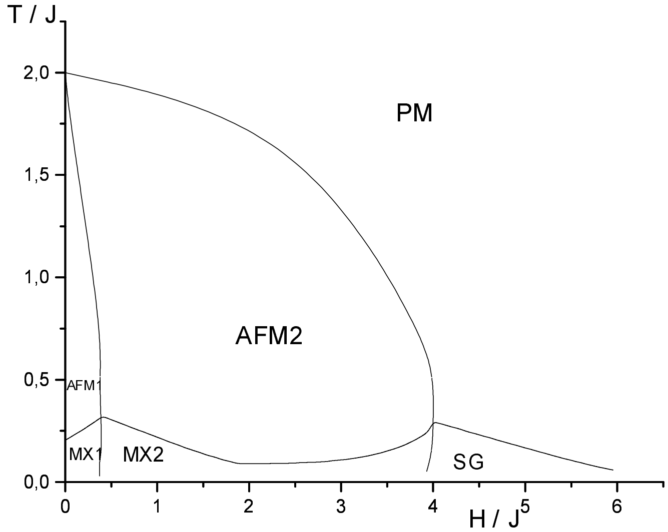

Fig. 2. Temperature *versus* field phase diagram for $p = 3$ and $J_0/J = 4.0$. The phases are: PM (paramagnetic), AFM1 (antiferromagnetic with all three sublattice magnetizations different), AFM2 (phase where two sublattices have equal magnetizations which is distinct from the third one), MX1 (mixed phase spin glass-AFM1), MX2 (mixed phase spin glass-AFM2), SG (spin glass).

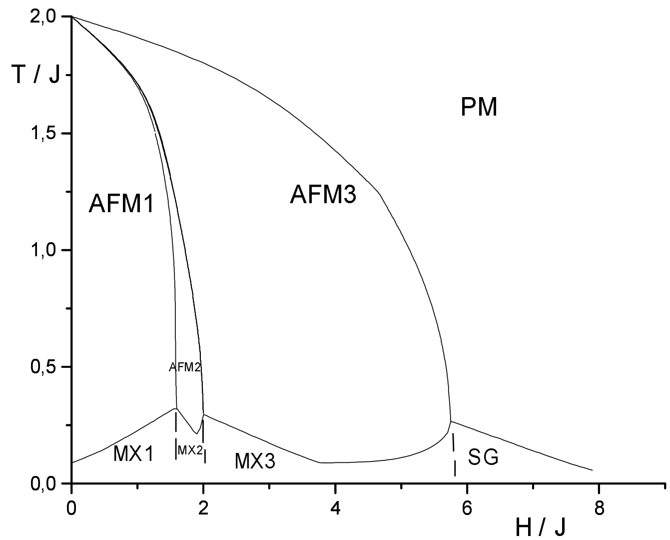

Fig. 4. Temperature *versus* field phase diagram for $p = 4$ and $J_0/J = 6.0$. The phases are: PM (paramagnetic), AFM1 (antiferromagnetic with pairs of sublattice magnetizations equal but distinct from the other pair), AFM2 (phase where two sublattices have equal magnetizations which is distinct from the others not equal two sublattice magnetizations), AFM3 (phase where three sublattices have equal magnetizations which is distinct from the fourth sublattice magnetization), MX1 (mixed phase spin glass-AFM1), MX2 (mixed phase spin glass-AFM2), MX3 (mixed phase spin glass-AFM3), SG (spin glass).

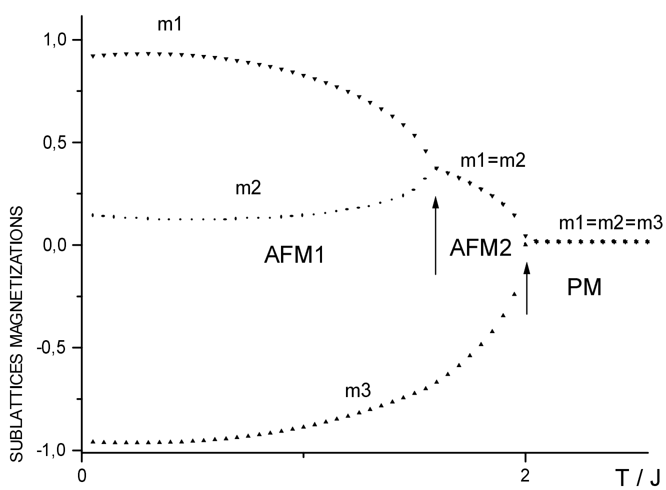

Fig. 3. Sublattice magnetizations as function of temperature for $p = 3$, $h = 0.1J$ and $J_0/J = 4.0$. The arrows indicate the transition temperatures between the present phases.

3.2 Non-zero fields

In non-zero external field, unlike an isotropic ferromagnet, for an antiferromagnet there is a line of Néel's phase transition $T_N(h)$ which may have continuous and first order transitions. A full determination of the phase diagrams for the present model requires to solve equations (9) and here we consider only the overall features of the model. In general, the multi-sublattices system may have p ergodic phases depending on the values of (T/J, J_0/J, h/J) and as many metastable states in the ergodic phase. Here, in a generalization of the cases $p = 1, 2$ [8,12], the non-ergodic phases spin-glass and mixed can unfold in p non-ergodic phases. Figure 2 shows the three-sublattices phase diagram for $J_0/J = 4$ where six phases occur: antiferromagnetic-1 (AFM1) characterized by having all sublattices magnetizations different; antiferromagnetic-2 (AFM2) where two sublattices have equal magnetizations which is distinct from the third one; paramagnetic phase where $m_1 = m_2 = m_3$; and the non-ergodic phases below the AT line: spin glass (SG), mixed spin glass-antiferromagnetic-1, 2 (MX1, MX2; the boundaries in this case are just a guide to the eye for even when $p = 2$ this has not been fully worked out). In Figure 3 the constant field ($h = 0.1J$) sublattice magnetizations as function of the temperature, exhibiting the phases changes, is shown. Figure 4 shows the four-sublattices phase diagram for $J_0/J = 6$ which has the phases: AFM1 ($m_1 = m_2 \neq m_3 = m_4$), AFM2 ($m_1 = m_2 \neq m_3 \neq m_4 \neq m_1$), AFM3 ($m_1 = m_2 = m_3 \neq m_4$), PM and the corresponding non-ergodic phases MX1, MX2, MX3 and SG. We have not shown in Figures 2, 3 and 4 the metastable regions (see below). Also, depending on the values of the parameters, some phase boundaries may have portions of continuous and first order transitions which we shall not study here.

The degree of disorder present (quantified by J_0/J) will determine how many phases are there and how large

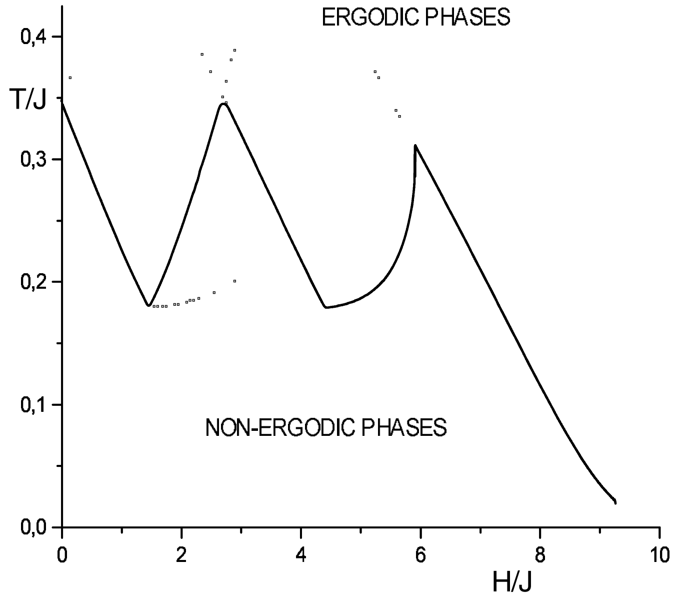

Fig. 5. Temperature *versus* field phase diagram for $p = 5$ and $J_0/J = 6.8$. It is shown only the AT-line for this case. The phases follow the same pattern as in previous cases. The dotted lines indicate the instability against RSB of metastable states.

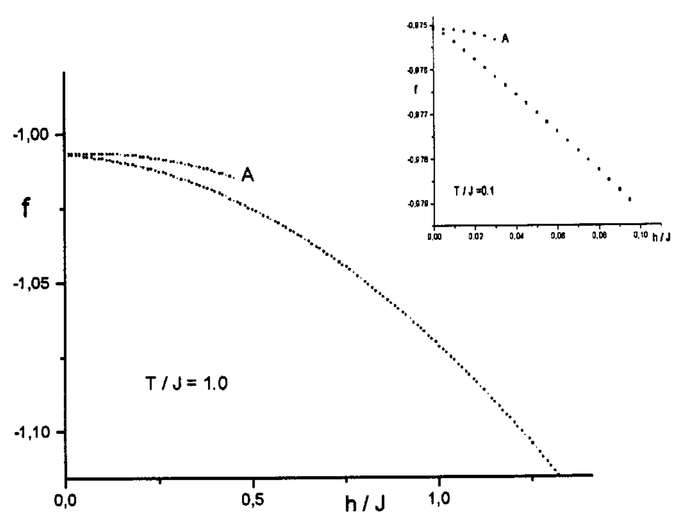

Fig. 6. Isothermal free energy at low fields for $T/J = 1.0$ and $T/J = 0.1$ (insert) for $p = 5$ and $J_0/J = 6.8$. The metastable branches (A) are shown.

they are in the phase diagram for given p. In Figure 5 it is shown the AT line for the five-sublattices system for $J_0/J = 6.8$ (multicritical point M in Fig. 1, in general, is at $T/J = 1.0$ and $J_0/J = p - 1$). For this case, some antiferromagnetic phases which exist for higher values of J_0/J have practically coalesced remaining strong metastability effects even for the ergodic solutions: the metastable solutions found in the ergodic phases may become unstable against RSB above or below the AT line. In other words, depending on the degree of disorder and the cooling way the system may be trapped into a ergodic state below the glassy transition! This kind of behavior seems to occur in diluted antiferromagnets. Figure 6 shows the behavior of the isothermal free energy equation (5) close to $h = 0$ displaying both the equilibrium and the metastable branches; the insert is for low temperature where both branches are unstable with respect to RSB. Notice that now there should exist (not worked out here) two physically distinct solutions not related by gauge symmetry.

From the present study, albeit incomplete, of the antiferromagnetic multisublattice ordering systems we can foresee that its extension to finite dimensions certainly will be very difficult for several reasons. Among others, the usual Néel critical point for the two-sublattices case will become a multicritical point, possibly of order $2p$, first-order transitions may appear (as for fcc ($p = 4$) lattices [23]) and a field theoretical approach may require a ϕ^{2p}-field theory to describe the phases transitions. In such a scenario, as many phases coalesce close to the multicritical point M in Figure 1, it is not clear what kind of field theory should be employed in its vicinity but certainly it has a much more rich structure than the Spin Glass-Ferromagnetic-Paramagnetic multicritical point [24].

In closing we would like to stress the mean field character of the present study and as such some features of the model may never occur in real systems. In principle, there is no reason why there could not be different phases both in the ergodic and in the non-ergodic regions. For instance, it is known that the inclusion of intrasublattice interactions (Fyodorov *et al.* (1990) [12]) explains satisfactorily some experimental results for some diluted metamagnets an extension which can be carried out for the present model (among others) from experimental motivation or from theoretical curiosity. Elsewhere [25] we have argued that the model here studied may account for the glassiness of four sublattices systems like some pyrochlores.

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